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Stationary Logic and Its Friends — I

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This is the first of two papers that deal with $L(aa)$ and related logics. Here we establish: every consistent $L_{\omega_1\omega}(Q)$ sentence has an $\underline{F}$-determinate model ($\underline{F}$ is a countable fragment of $L_{\omega_1\omega}(aa)$); and it is consistent that $L(Q)$ has the weak Beth property. A logic has the weak Beth property if it satisfies Beth's theorem where the hypothesis has been strengthened to require that implicit definitions guarantee existence as well as uniqueness. In [4] Friedman showed that Beth's theorem fails for $L(Q)$. He asked whether $L(Q)$ has the weak Beth property. (This is also problem 8 in [5].) Friedman has argued that people were interested in the weak Beth property and the usual theorems of Beth and Craig just happen to be true (for $L_{\omega\omega}$).

The two sections of this paper can be read independently. The methodological link between the sections is the use of forcing (set theoretic rather than model theoretic) to construct models. How does forcing help us? In the model theoretic proofs of the theorems of Beth and Craig, saturated models are used. Mainly one uses that these models have lots of automorphisms. Such models are harder (or impossible) to come by for other logics. Forcing can be viewed as giving Boolean-valued models. So we can use automorphisms which also move truth values. Sometimes by using the completeness theorem (or more generally absoluteness arguments) we can get rid of the forcing.

In the second paper we will use our methods to investigate the relation between $L(aa)$ and other logics. In particular we'll show it is consistent that $\Delta(L(Q)) \subseteq L(aa)$; Craig ($L(Q_\omega^{cf})$, $L(aa)$) holds (Q_ω^{cf} expresses "the cofinality of a linear order is ω"); and there is a compact Beth closed logic stronger than $L_{\omega\omega}$.[1] These results should be viewed against a background of counterexamples

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