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Homogeneous Boolean Algebras with Very Nonsymmetric Subalgebras

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We prove the following theorems.

Theorem 1 *For every Boolean algebra A there are extensions $C \supseteq B \supseteq A$ such that B and C are homogeneous, every endomorphism or automorphism of A extends to an endomorphism or automorphism of B, and no nontrivial one-one endomorphism of B extends to an endomorphism of C.*

Theorem 2 *Assume ($\diamond$). There is an ω_1-Souslin tree T such that the regular open algebra B of T is homogeneous and has a complete subalgebra A onto which no nontrivial automorphism of B restricts.*

These theorems were motivated by the following question raised by Štěpánek: Does there exist a complete homogeneous Boolean algebra B with a complete homogeneous subalgebra A such that no nontrivial automorphism of A extends to B? Here a Boolean algebra B is called homogeneous if every principal ideal $B \restriction b = \{x \in B \mid x \leqslant b\}$ for $b \neq 0$ is isomorphic to B; because of $B \cong B \restriction b \times B \restriction -b$, $B \restriction b$ is also called a factor of B. B is said to be rigid if it has no nontrivial automorphism.

Štěpánek's question arose from the following facts. Every Boolean algebra A can be embedded into a homogeneous complete algebra B such that every automorphism of A extends to B (see [4] and [5]). Every A can be embedded into a complete rigid B—of course, no nontrivial automorphism of A extends to B (see [7]). Every A can be embedded into a complete B without homogeneous or rigid factors such that either every or no nontrivial automorphism of A extends to B (see [8] and [9]).

We assume acquaintance with [6] for the proof of Theorem 1 and with [1] or [3] for Theorem 2.

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