Notre Dame Journal of Formal Logic
Volume 23, Number 4, October 1982

Axioms for Tense Logic
II. Time Periods

JOHN P. BURGESS

The latest fashion in tense logic is for systems based on time periods rather than durationless instants. The present note provides an axiomatizability result for the period-based tense logic of the rationals and the reals, inspired by the work of P. Röper [1].

1 Structures

1.1 Instant-based case Here we work with structures $\mathcal{X} = (X, <)$ where X is a nonempty set, $<$ a binary relation on X. Intuitively, X represents the set of instants of time, and $<$ the earlier/later relation. In the present note we will consider only those $\mathcal{X}$ that are dense linear orders without first or last element. This of course takes in the usual orders on the rational and real numbers, denoted $\mathcal{Q}$ and $\mathcal{R}$, respectively. Let $\mathcal{K}$ be the class of all such orders. For $\mathcal{X} = (X, <) \in \mathcal{K}$ the order relation $<$ on X determines also a topology on X, having as basis the open intervals $]x,y[\ = \{z: x < z < y\}$ of $\mathcal{X}$. Thus such topological notions as *regular open set* and *nowhere dense set* can be applied to subsets of X.

1.2 Period-based case Here we work with structures $\mathcal{Y} = (Y, \subseteq, \lhd)$ where Y is a nonempty set, $\subseteq$ and $\lhd$ binary relations on Y. Intuitively, Y represents the set of all nonempty finite uninterrupted periods of time, and $\subseteq$ and $\lhd$ the inclusion and earlier/later relations among such periods. For $\mathcal{X} = (X, <) \in \mathcal{K}$ we introduce the structure $I(\mathcal{X}) = \mathcal{Y} = (Y, \subseteq, \lhd)$ given by:

Y = the set of nonempty open intervals $]x,y[$ of $\mathcal{X}$
$\subseteq$ = the usual set-theoretic inclusion relation
$\lhd$ = the natural order relation induced by $<$, namely:

$$]x,y[\ \lhd\ ]z,w[\text{ iff } y \leqslant z.$$

Received April 3, 1981