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The Modal Status of Antinomies

GERT-JAN C. LOKHORST*

What is the modal status of antinomies?[1] Classical modal logic provides no interesting answer to this question because it lets antinomies turn all well-formed formulas (including all modal formulas) into theorems. In the present note we propose two nonclassical modal systems which do not suffer from this defect. Both systems are obtained by supplementing the semantics of Asenjo's and Tamburino's antinomic propositional logic L (see [1], familiarity with which will be assumed in this article) with a very natural-sounding truth condition for modal formulas. The surprising result is that antinomies are in any case both *necessary* and *impossible*: according to the second system we propose, they are both *non-necessary* and *possible* as well. It may be doubted whether these results are in accord with our intuitions. However, it should be remembered that our intuitions were formed during centuries of classical slumber; acquiring the right intuitions in antinomic thinking may simply be a matter of time.

1 The systems

1.1 Language The language is as in [1], p. 19, but add to formation rule 2: if $\mathcal{B}_1$ is a statement form, $\Box\mathcal{B}_1$ is a statement form. Definitions:

$$\neg^*\mathcal{B}_1 =_{df} \mathcal{B}_1 \supset A_1 \;\&\; \neg A_1;\; \Diamond\mathcal{B}_1 =_{df} \neg\Box\neg\mathcal{B}_1.$$

1.2 Semantics An antinomic model is a triple $\langle W, R, V\rangle$, where W is a set (of "possible worlds"), $R \subseteq W \times W$, and V: AT $\times W \rightarrow \{0,1,2\}$. (Here AT is the set of atomic statements.) $V(A_i, w) = 0$ or 1, whereas $V(B_i, w) = 2$.

The interpretation function I is defined as follows:

1. $I(A_i, w) = V(A_i, w)$, $I(B_i, w) = V(B_i, w)$.
2. $I(\neg\mathcal{B}_1, w)$, $I(\mathcal{B}_1 \$ \mathcal{B}_2, w)$, where \$ is a truth-functional connective: as given in the tables in [1], p. 18, suitably relativized to the world w.

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