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Constructive Predicate Logic with Strong Negation and Model Theory

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In this paper, we attempt to investigate the so-called constructive predicate logic with strong negation from a model-theoretical point of view.

Strong negation was first introduced by Nelson [8] in connection with Kleene's *recursive realizability*. At the same time, Markov [7] showed independently that intuitionistic (Heyting) negation can be defined by strong negation and implication. Then Vorob'ev [14] formulated constructive propositional logic with strong negation. Polish logicians such as Rasiowa [9],[10] studied in terms of lattice theory. Additionally, we can find a Gentzen-type formulation by Almukdad and Nelson [1] and Ishimoto [5],[6], and a model-theoretic study by Thomason [13] and Routley [11], to cite only a few.

Strong negation is a constructive negation different from *Heyting negation*. For example, in intuitionistic logic, $\neg(A \wedge B)$ is not equivalent to the derivability of at least one formula of $\neg A$ or $\neg B$. And we cannot prove the equivalence between $\neg\forall xA(x)$ and $\neg A(t)$. But these are equivalent in constructive logic with strong negation.

In this paper, we research this system on the basis of Kripke's *many worlds semantics* (the so-called *Kripke model*), and try to provide a *Henkin-type proof* of the completeness theorem for the system. We will also inquire into the relationships among constructive predicate logic with strong negation, classical logic, and intuitionistic logic.

1 Constructive predicate logic with strong negation Constructive predicate logic with strong negation, instead of Heyting negation, is designated as S. As stated above, Heyting negation can be defined in S by way of strong negation and implication, as S does not have it as one of its primitives. We call the sys-

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