

Variations on a Thesis: Intuitionism and Computability

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1 Prelude The theme of this paper is Church's Thesis (or CT) as it is normally understood by intuitionists and by logicians concerned with constructivity. This is to be distinguished from the more familiar "quasi-empirical" statement of the same name — that

every mechanically computable function is general recursive.

Rather, we will use CT to denote one or another version of the intuitionistic mathematical statement that

every total natural number function is general recursive.

There are three variations. The first is an extended argument for a reappraisal of the status of CT within intuitionism. Traditionally, the intuitionists' attitude toward CT has been strongly negative; it was thought that Church's Thesis was obviously false. The fact that it is consistent with the main bulk of constructive mathematics was either to be deplored or ignored.

We think this attitude unfortunate. As it seems to derive a good part of its impetus from an unnecessary identification of intuitionism with *reductive intuitionism*, we devote the bulk of the first variation to suggesting that reductive intuitionism might itself be either deplored or ignored.

Of course, the idea that CT is obviously false can be reinforced from other quarters. Logicians often appeal to a kind of hypothetico-deductivism and argue that CT is false because its mathematical consequences are largely untoward. The

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