

## Modal Trees: Correction to a Decision Procedure for $S5$ (and $T$ )

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Every now and then, this *Journal* has published extensions of the tree method given by Jeffrey in [5] to yield decision procedures for propositional calculi other than classical. This has been done, for instance, in [3] for modal logics  $T$  and  $S5$ , and in [1] for Lemmon's minimal tense logic  $K_t$ . In this paper, our main interest is focused on modal trees for  $S5$ ; the attention paid by us here to modal trees for  $T$  is only of secondary importance. The existence of effective decision methods for both systems is well known from long ago (e.g., see [4]). The tree method of [5] being the simplest and most elegant decision procedure for classical propositional calculus, extending it in order to cover these propositional modal logics with a similar degree of simplicity is therefore desirable.

Unfortunately, the well conceived attempt made in [3] by Davidson, Jackson, and Pargetter fails to yield a successful decision procedure for  $S5$ , as we aim to show here. Davidson seems not to be conscious of the fault, since in [2] (a more recent paper) she repeats the same failure with regard to a testing procedure for modal trees for modal predicate logics. It seems that Jackson and Pargetter have not seen the point either, as is revealed in a footnote on p. 56 of [2], where Davidson declares her indebtedness to them for their remarks and suggestions.

However, a quite successful decision procedure can be easily obtained by a small (although decisive) correction to the above procedures, as we are also going to show later. In doing this, we will be applying the strategy of Copeland in [1] for the analogous treatment of tense trees. In [3], the procedure is designed for  $T$  as well as for  $S5$ ; applied to  $T$  it is in fact an effective decision procedure, in contrast to the case of  $S5$ . Our correction is designed in turn in such a way that it can also be applied to  $T$ , though in this case only a more elegant decision procedure, in the sense of yielding shorter trees in some cases, is to be obtained.

To begin, we summarize the modal rules of the tree method of [3] as follows (where  $\phi$  is a wff, and  $\phi^i$  is the result of indexing all its propositional letters with the same superscript  $i$ ):

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