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Connection Structures

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Abstract B. L. Clarke, following a proposal of A. N. Whitehead, presents an axiomatized calculus of individuals based on a primitive predicate "x is connected with y". In this article we show that a proper subset of Clarke's system of axioms characterizes the complete orthocomplemented lattices, while the whole of Clarke's system characterizes the complete atomless Boolean algebras.

1 Introduction In [2] and [3] Clarke presents an axiomatized calculus of individuals based on a primitive predicate "x is connected with y". Such a calculus represents a revised version of the proposal made by Whitehead in *Process and Reality* and is similar to the calculus proposed by Leonard and Goodman in [5].

In this article we show that a proper subset of Clarke's system of axioms characterizes the complete orthocomplemented lattices, while the whole of Clarke's system characterizes the complete atomless Boolean algebras.

2 Connection structures Let R be a nonempty set and C a binary relation on R, set $C(x) = \{y \in R/xCy\}$ and suppose the following axioms are true of every $x,y \in R$:

A1 xCx;
A2 $xCy \Rightarrow yCx$;
A3 $C(x) = C(y) \Rightarrow x = y$.

We call *regions* the elements of R and, if $x,y \in R$ and xCy, we say that x is *connected* with y. If X is a nonempty subset of R, we say that x is the *fusion* of X just in case for every $y \in R$, xCy iff for some $z \in X$, zCy; in other words, x is the fusion of x provided that

$$C(x) = \cup\{C(z)/z \in X\}. \tag{1}$$

The fusion of the nonempty subsets of R is assured by the following axiom.

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