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Isomorphic but Not Lower Base-Isomorphic Cylindric Algebras of Finite Dimension

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Abstract This article deals with Serény's theorem giving sufficient conditions for two cylindric set algebras (*Cs*'s) to be lower base-isomorphic, a cylindric algebra version of Vaught's theorem on the existence of prime models of atomic theories in countable languages. It is proved that Serény's theorem requires all the conditions given in its statement. Here the necessity of the condition of the infinite-dimensionality of the given *Cs*'s is proved via constructing isomorphic but not lower base-isomorphic *Cs*'s of any finite dimension greater than one. A model-theoretical corollary of the above dependence is stated also.

In this paper we will prove that the (cylindric) algebraic version of Vaught's theorem concerning the existence of prime models of atomic theories does not hold for finite dimensional cylindric set algebras, i.e. for algebras corresponding to models for languages with finitely many variable symbols, partly solving a problem posed in [4] and [9]. Let us see this statement in a little more detail. The algebraic version of the Vaught theorem referred to above is the following theorem of Serény [10]: *Any isomorphism between two infinite dimensional countably generated regular and locally finite Cs's with atomic neat n-reducts (for all finite n) is a lower base-isomorphism.* (The notation used in this theorem is defined below, and also in [6] and [7].)

We will show that the condition that the algebras concerned be of infinite dimension cannot be dropped. It is worth adding that we have already proved this fact about all other conditions in this theorem (cf. [2], [3], and [4]). Our treatment is based on [6] and [7]; consequently, we follow the terminology and notation of these volumes except that we denote the full Cs_α with base U by

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