

Book Review

Penelope Maddy. *Naturalism in Mathematics*. Clarendon Press, Oxford, 1997. viii + 254 pages.

1 Theory choice in set theory Set theorists investigate sets. They also investigate theories. Set theorists investigate certain formal theories because:

1. their study of those theories teaches them interesting things about their own informal set theorizing; and
2. the metatheory of those theories is mathematically rich and interesting in its own right.

They get (1), of course, because theories like ZFC are successful formalizations of informal set theorizing. But they get (2) partly because certain features of informal set theorizing are omitted from the formalizations. Set theorists speaking a dialect of English can, for example, fully characterize the order type of the natural numbers. But it is the formalizations that *cannot* characterize ω categorically that are most attractive metamathematically. So there is a tradeoff: we get the most bountiful metatheory only if we concentrate on theories too weak to capture every aspect of informal set theorizing. First-order languages lack the full expressive capacity of informal mathematical discourse, but they have a metatheory that many mathematicians find stimulating and suggestive and entertaining and useful and generally wonderful. So three cheers for first-order languages! Let us just not forget their limitations.

The formal set theory of our dreams would have a great metatheory *and* would fully capture every important aspect of informal set theorizing. Well (consarnit!) there could be a perfect fit between the formal and the informal if the set theorists would limit their *informal* discourse. If only they would steer clear of those pesky informal locutions that are not firstorderizable! If only we could *make* them speak “an austere sublanguage of English that corresponds to the limited resources of first-order logic” (p. 212). Of course, we couldn’t do so even if we tried. And if we did try, we ourselves would violate a fundamental principle of philosophical methodology and we would be urging the set theorists to erase a fundamental principle of set theoretic methodology.

The philosophical principle is MODESTY: “. . . philosophy is not in the business of criticizing and recommending reform of good mathematics on extra-mathe-