Notre Dame Journal of Formal Logic
Volume VII, Number 1, January 1966

A CALCULUS OF ANTINOMIES

F. G. ASENJO

1. *Truth tables for antinomies.* Let us assume that atomic propositions have either one or two truth values. Propositions (atomic or compound) will then be either true, false, or true and false. Let us call those propositions which are both true and false "antinomies." Our immediate purpose is to extend the classical propositional calculus to include operations with antinomies. Therefore, we take the truth tables for the five classical propositional connectives and add the following rule for operations that involve antinomies. The truth value or values of $A \tau B$ (where τ stands for any binary connective) are the value or values that result from giving A and B all possible combinations of the truth values. If A and B have only one value, then the classical pattern follows. But if A or B, or both, are antinomies, then the value or values of $A \tau B$ depend on the values that are obtained when A and B assume all their different truth values in succession. For example, the following case arises for $A \supset B$: (a) A antinomic, B true, then $A \supset B$ shall be considered true, since $A \supset B$ is true whatever the truth value of A; (b) A false, B antinomic, then $A \supset B$ will be true for a similar reason. By indicating true, false, and antinomic with the symbols *0, 1, 2,* respectively, the entire situation can be described with these tables.

$A \supset B$

A \ B	0	1	2
0	0	1	2
1	0	0	0
2	0	2	2

$A \& B$

A \ B	0	1	2
0	0	1	2
1	1	1	1
2	2	1	2

$A \vee B$

A \ B	0	1	2
0	0	0	0
1	0	1	2
2	0	2	2

$A \sim B$

A \ B	0	1	2
0	0	1	2
1	1	0	2
2	2	2	2

$\neg A$

A	$\neg A$
0	1
1	0
2	2

If one applies the tables in the order in which a compound proposition is generated, it is possible to assign one of the three truth values to any well-formed proposition. Three different cases arise, depending on the domain of atomic propositions to which the five operations are to be applied. The atomic propositions can be (1) all single-valued, (2) all antinomic, or (3) some single-valued and some antinomic. Case 1 is the

Received June 20, 1964