454
Notre Dame Journal of Formal Logic
Volume XII, Number 4, October 1971
NDJFAM

COPI'S METHOD OF DEDUCTION AGAIN

M. C. BRADLEY

Professor Copi, in the recent 3rd edition of *Symbolic Logic* [1], leaves unchanged the adaptation of Canty's proof of the completeness of **CMD** [2] which he used in the 2nd edition. The object of the present note is twofold. (1) To establish a lemma of the proof which neither author explicitly establishes, and to show that in view of the way in which this lemma needs to be established, the Copi-Canty proof involves a pointless complication. (2) To establish another lemma of the proof, namely that the replacement rules of **CMD** are adequate to deriving from any line in a proof its DNF. The literature, of course, contains various proofs to the effect that any propositional formula can be reduced to DNF within some version of propositional logic. What is proposed here is another proof to the same effect, relating specifically to **CMD**.

(1) Each author uses Metatheorem A below. A proof is supplied.

Metatheorem A: *If* $P_1^s, P_2^s, \ldots, P_n^s \therefore Q^s$ *is a valid argument whose validity depends solely on truth-functional considerations, then* $P_1, P_2, \ldots, P_n \vdash Q$ *in* **RS**.[1]

Proof: In **RS** we can always construct the sequence Σ of wffs $P_1, P_2, \ldots, P_n, Q$. If this sequence can be enlarged to form a sequence $S_1, S_2, \ldots, S_k$, such that every S_i $(1 \leqslant i \leqslant k)$ is either a P_i $(1 \leqslant i \leqslant n)$, or an axiom of **RS**, or is derived from two preceding lines of the sequence by the use of R1, and S_k is Q, then $P_1, P_2, \ldots, P_n \vdash Q$. Now by hypothesis $P_1^s, P_2^s, \ldots, P_n^s \therefore Q^s$ is a truth-functionally valid argument, and therefore $(P_1^s \cdot P_2^s \cdot \ldots \cdot P_n^s) \supset Q^s$ is tautologous. Thus, by the completeness of **RS**, $\vdash (P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q$. Thus there is a sequence Σ' of wffs of **RS**, such that every wff is an axiom of **RS** or follows from two preceding wffs by R1, the last wff of which is $(P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q$. Suppose Σ' prefaced to the sequence Σ to form the sequence Σ''. Σ'' can then be enlarged to from Σ''', where Σ''' is a sequence $S_1, S_2, \ldots, S_k$, as follows.

---

1. Where the P_i^s's and Q^s are the P_i's and Q as interpreted (normally) within a semantical system $\mathcal{S}$.

Received September 17, 1969