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A NOTE ON UNIVERSALLY FREE DESCRIPTION THEORY

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1 In this note a universally free first order logic—(UFL) for short—in the sense of Meyer and Lambert [5], incorporating the rubrics required for a treatment of "descriptions" will be sketched. (UFL) is intended to satisfy the following conditions: (a) the self-identity of terms should be assertable in (UFL) without qualifications or restrictions; (b) i) $(\forall_{x_i})\phi_i \rightarrow (\exists_{x_i})\phi_i$, and ii) $(\forall_{x_i})\phi_i \rightarrow (S_{x_i})\phi_i$—where (S_{x_i}) is the singular quantifier to be read as 'there is only one . . . such that'—should not be assertable in (UFL); (c) Quine's criterion of ontological commitment, as interpreted in Rao [11], should be applicable to theories incorporating (UFL).

2 Let (SFL) be the standard system of first order logic, as presented by, for instance, Mendelson [6]. To have (UFL) we shall (1) augment the primitive base of (SFL) by i) some specially introduced *monadic predicate constants* A^i, $i = 1, 2, \ldots$, as in Rao [9], such that A^i_J, $i \leqslant J$ is a wff of (UFL), and ii) the singular quantifier $(S\ \ldots)$, where the blank is to be filled by an individual variable, such that if ϕ_i is a wff of (UFL) containing free occurrences of an individual variable x_i, $(S_{x_i})\phi_i$ is a wff of (UFL); and (2) delete from the primitive base of (SFL) the equality sign =. (This deletion is motivated by considerations shown in Rao [10].) To pick up the theorems of (UFL), we shall replace the meta-axioms, and rules of (SFL) by the following:

Ax1. *If* ϕ_i *is a tautology by two-valued truth tables then* $\vdash\phi_i$.
Ax2. $\vdash\ulcorner(\forall_{x_i})(\phi_i \rightarrow \phi_J) \rightarrow ((\forall_{x_i})\phi_i \rightarrow (\forall_{x_i})\phi_J)\urcorner$.
Ax3. $\vdash\ulcorner\phi_i \rightarrow (\forall_{x_i})\phi_i$ *provided* x_i *does not occur free in* ϕ_i.
Ax4. $\vdash\ulcorner(\forall_{x_i})\phi_i \rightarrow \phi_i\urcorner$ *provided* x_i *does occur free in* ϕ_i.
Ax5. $\vdash(\forall_{x_i})A^J_{x_i}$ *provided* $J = i$.
Ax6. $\vdash\ulcorner(A^i_{x_J} \rightarrow A^J_{x_i}) \rightarrow (\phi_i \rightarrow \phi_J)$ *provided* ϕ_i *and* ϕ_J *are alike except that* ϕ_J *contains* x_i *wherever* ϕ_i *contains* x_J.
Ax7. $\vdash\ulcorner(\exists_{x_i})\phi_i \rightarrow ((\forall_{x_J})\phi_J \rightarrow (S_{x_i})\phi_i)\urcorner$ *where* ϕ_i *and* ϕ_J *are alike except that* ϕ_i *contains* x_i *wherever* ϕ_J *contains* x_J, *and* $i \leqslant J$.
Ax8. $\vdash\ulcorner(S_{x_i})\phi_i \rightarrow ((\exists_{x_i})\phi_i \rightarrow ((\forall_{x_i})\phi_J \rightarrow (A^i_{x_J} \rightarrow A^J_{x_i})))\urcorner$ *provided* ϕ_i *and* ϕ_J *are as in* Ax7.
Ax9. *If* $\vdash\phi_i$ *and* $\vdash\ulcorner\phi_i \rightarrow \phi_J\urcorner$ *then* $\vdash\phi_J$.

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