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ALTERNATIVE NOTATIONS FOR *PRINCIPIA MATHEMATICA* DESCRIPTION THEORY: POSSIBLE MODIFICATIONS

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1 The following are formulas by clauses (1)-(7), pp. 64-65, of a recent paper:[1]

$$[\mathbf{\iota} y H^1 y] I^2 x \mathbf{\iota} y H^1 y$$
$$[\mathbf{\iota} y J^1 y]\ [\mathbf{\iota} x H^1 x] I^2 \mathbf{\iota} y J^1 y \mathbf{\iota} x H^1 x$$
$$\wedge x [\mathbf{\iota} y H^1 y] I^2 x \mathbf{\iota} y H^1 y$$
$$[\mathbf{\iota} y H^1 y] \wedge x I^2 x \mathbf{\iota} y H^1 y$$

But the following are *not* formulas by these clauses:

$$[\mathbf{\iota} x H^1 x] I^2 x \mathbf{\iota} x H^1 x$$
$$[\mathbf{\iota} x J^1 x]\ [\mathbf{\iota} x H^1 x] I^2 \mathbf{\iota} x J^1 x \mathbf{\iota} x H^1 x$$
$$\wedge x [\mathbf{\iota} x H^1 x] I^2 x \mathbf{\iota} x H^1 x$$
$$[\mathbf{\iota} x H^1 x] \wedge x I^2 x \mathbf{\iota} x H^1 x$$

A connected point is that, by translation rules $\bar{\mathbf{T}}/\mathbf{\iota}$ and $\mathbf{\iota}/\bar{\mathbf{T}}$, not only is ϕ' a translation of ϕ by $\mathbf{\iota}/\bar{\mathbf{T}}$ if and only if ϕ is a translation of ϕ' by $\bar{\mathbf{T}}/\mathbf{\iota}$, but each $\mathbf{\iota}$-formula has a unique $\mathbf{\iota}$-free $\bar{\mathbf{T}}$-translation and vice versa.

Modifications to formation and translation rules are possible, and are given below, that secure as formulas all of the above strings (which may seem a gain) while trading the *unique*-translation feature for a *multiple*-translation feature (which may seem a loss).

2 Replace clause (7) by the following clause (7′):

(a′) An expression $\mathbf{\iota}\alpha\phi$, α a variable and ϕ a formula or pseudo-formula, is an $\mathbf{\iota}$-description.
(b′) An expression ϕ is a *pseudo-term* (*pseudo-formula*) just in case a term (formula) ϕ' is like ϕ except for having, in place of all occurrences in ϕ of one or more $\mathbf{\iota}$-descriptions, occurrences of variables not in ϕ. A term (formula) related to a pseudo-term (pseudo-formula) ϕ in this manner is an *associated term* (*formula*) of ϕ.
(c′) *An occurrence of a variable α is bound* in a term or formula π just in case it stands within an occurrence in π of an expression χ such that (i) either χ is $\wedge\alpha\phi$, $\vee\alpha\phi$, $\urcorner\alpha\phi$, or $\bar{\mathbf{T}}\alpha\phi\psi$, or χ is $[\mathbf{\iota}\alpha\phi]\psi$ and the occurrence of

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