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Concerning Some Cylindric Algebra Versions of the Downward Löwenheim-Skolem Theorem

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1 Introduction The theory of cylindric algebras (CA's) is the algebraic theory of first-order logics. Several ideas about logic are easier to formulate in the frame of CA theory. Examples are some concepts of Abstract Model Theory (cf. [1]–[4], [8], [11]–[13], [22], and [24]), as well as considerations about relationships between several axiomatic theories of different similarity types (cf. [5]–[7], [9], [14], and [22]). (This second topic is sometimes mentioned under the name "Theory Morphisms", "the category of theories and theory morphisms", or "interpretations".) For these reasons, certain branches of theoretical computer science are based on algebraic logic instead of pure logic (cf. e.g. [9], [14]). For applications of CA theory in computer science, see e.g. [9], [10], [15], [20], and [21].

The connection between logic and CA theory is elaborated in [1], [3], [11], [12], [16], [19], and [22]. The connection between model theory and CA theory is elaborated in [19], [22], [23]. For example, it is proved in [23] that the simple algebraic property of a class K of CA's that all epimorphisms in K are surjective is equivalent to a definability theoretic property of first-order logics (more precisely, model theories) associated to K.

It was found that in general it is the classes Crs_α and Gs_α^{reg} of cylindric set algebras that provide the fundamental link between model theory and CA theory. The CA-theoretic counterparts of the model theoretic notions are usually the fundamental notions of Crs_α (and Gs_α^{reg}) theory. (It is shown in [22] that Crs_α's which are not Gs_α^{reg}'s arise from nonclassical and unusual model theories when the usual process of algebraization is applied to them.) CA theory is much more "algebraic abstract model theory" than "algebraic classical first-order logic". This helps to explain the fact that frequently CA counterparts of classical results are harder to prove than those results: the CA counterparts say that

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