

## The Theory of Descriptions Revisited

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An *excursus* is carried out through the principal steps in the development of the theory of descriptions (TD) from B. Russell until now, and its most important advantages and disadvantages are sketched. TD is studied in the context of model theory (in A. Robinson's style), taking preservation and classification theorems based on normal forms into consideration. Finally, the categorical formulation of TD in topos theory, starting from M. Fourman and D. Scott, is presented with reference to sheaves.

**1 Introduction** A 'descriptive' operator is a function  $\Delta$  which, given an open formula  $\phi(x)$  of a language  $L$  as its input, gives an  $L$ -term  $\Delta x\phi(x)$ , called a 'description', as its output. (In the following,  $L$  is supposed to be a standard first-order language with identity.) Clearly, there are many such functions, and to account for at least some of them is a project not only of mathematical significance but also with extensive application to linguistics, because natural languages present a vast range of problems involving the articles "the" and "a"; the theory of descriptions (TD) concerns set theory, with the abstraction operator  $\{ | \dots \}$ , and recursion theory, from  $\mu$ -operators to  $\lambda$ -terms. Here I shall limit myself to a general consideration of *definite* descriptions, i.e. singular terms generated by  $\Delta$ 's which can be read "the such and such", denoted by  $|x\phi(x)$ .

The difficulties met within many contemporary attempts to formalize descriptions lead to approaching the problem in an unusual way, based on the intuition that the kernel of TD is the *presence (or absence) of symmetries* in a universe of discourse, and therefore in semantics. So, given a model  $\mathfrak{M}$  for a theory  $T$  in  $L$ , we shall focus on the class  $Aut_A(\mathfrak{M})$  of  $\mathfrak{M}$ -automorphisms pointwise (for simplicity) fixed on a set  $A \subseteq \mathfrak{N}$ , where  $\mathfrak{M}$  is a substructure of  $\mathfrak{N}$ , in order to classify descriptions obtained through parameters from  $A$ . As  $A$  is varying, this classification proves to be strictly related both to philosophical and

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