

## A Constructivism Based on Classical Truth

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**1 Introduction** Our aim, in this paper, is to study the relationships between constructive truth and classical truth under a unified point of view, namely, to analyze the properties of *constructive* systems whose language is enriched by a sentential operator **T** assumed to represent the concept of *classical* truth. By a “constructive system” we mean a system  $S$  in which if  $\vdash_S A \vee B$  then  $\vdash_S A$  or  $\vdash_S B$  and if  $\vdash_S \exists x A$  then  $\vdash_S A[t/x]$ , for some term  $t$ . We say that an operator **T** represents in  $S$  the concept of classical truth if

$$(1) \quad \vdash_{\text{CPrC}} A \text{ iff } \vdash_S \mathbf{T}A$$

where CPrC is classical predicate calculus (we will also use “CPC”, “IPC”, and “IPrC” to denote, respectively, classical propositional calculus, intuitionistic propositional calculus, and intuitionistic predicate calculus).

The importance of this study should be evident to whoever is interested not in reducing classical truth to constructive notions or vice versa, but in determining the laws of interaction of these notions.

For brevity's sake we shall limit our exposition to the propositional fragments of our calculi, giving some indications about their predicative extensions in notes.

In Section 2 we will introduce the calculus  $E_0$  and its Kripke semantics; in Section 3 we will deal with the problem of finding *maximal* constructive systems in which **T** represents classical truth and we will prove a theorem of maximality for a calculus we call  $E^*$ . For this calculus we will introduce a kind of semantics based on the notion of “valuation form”, and its specific features will be discussed in Section 3.

At this point, let us look at some problems connected with the introduction of an operator **T** in a constructive setting.

As IPrC is the most famous system and, let us say, the paradigmatic example of a constructive system, the most obvious way to reach our aim would seem to consist of adding an operator **T** to IPrC and then characterizing it by suit-