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ON THE PERMANENTS OF SOME TRIDIAGONAL MATRICES WITH APPLICATIONS TO THE FIBONACCI AND LUCAS NUMBERS

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ABSTRACT. In this paper, we derive some interesting relationships between the permanents of some tridiagonal matrices with applications to the negatively and positively subscripted usual Fibonacci and Lucas numbers. Also, we give a relation involving the generalized order-k Lucas number and permanent of a matrix.

1. Introduction. The Fibonacci sequence, $\{F_n\}$, is defined by the recurrence relation, for $n \geq 1$

$$F_{n+1} = F_n + F_{n-1} \tag{1.1}$$

where $F_0 = 0$, $F_1 = 1$. The Lucas sequence, $\{L_n\}$, is defined by the recurrence relation, for $n \geq 1$

$$L_{n+1} = L_n + L_{n-1} \tag{1.2}$$

where $L_0 = 2$, $L_1 = 1$.

Rules (1.1) and (1.2) can be used to extend the sequences backward, respectively, thus

$$\begin{aligned} F_{-1} &= F_1 - F_0, & F_{-2} &= F_0 - F_{-1} \\ L_{-1} &= L_1 - L_{0,} & L_{-2} &= L_0 - L_{-1}, \dots , \end{aligned}$$

and so on. Clearly,

$$F_{-n} = F_{-n+2} - F_{-n+1} = (-1)^{n+1} F_n, \tag{1.3}$$

$$L_{-n} = L_{-n+2} - L_{-n+1} = (-1)^n L_n. \tag{1.4}$$

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