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SEQUENCES OF CONSECUTIVE HAPPY NUMBERS

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1. Introduction. Let $S_2 : \mathbf{Z}^+ \to \mathbf{Z}^+$ denote the function that takes a positive integer to the sum of the squares of its decimal digits. A *happy number* is a positive integer a such that $S_2^m(a) = 1$ for some $m \geq 0$. In [**2**], happy numbers were generalized as follows: For $e \geq 2$, $b \geq 2$, and $0 \leq a_i < b$, define $S_{e,b} : \mathbf{Z}^+ \to \mathbf{Z}^+$ by

$$S_{e,b}\left(\sum_{i=0}^{n} a_i b^i\right) = \sum_{i=0}^{n} a_i^e.$$

If $S_{e,b}^m(a) = 1$ for some $m \geq 0$, then a is an *e-power b-happy number.*

Using a computer search, it is easy to find examples of short sequences of consecutive happy numbers. The least examples of sequences of lengths 1–5 are given in Table 1. A natural question to ask is whether or not there exist arbitrarily long finite sequences of consecutive happy numbers. In 2000, El-Sedy and Siksek [**1**] showed that the answer is yes.

One can also ask more generally for what values of e and b do there exist arbitrarily long sequences of consecutive e-power b-happy numbers. Some results are already known. For example, for all $e \geq 2$,

TABLE 1. CONSECUTIVE HAPPY NUMBERS.

Length	Least happy number sequence
1	1
2	31, 32
3	1880, 1881, 1882
4	7839, 7840, 7841, 7842
5	44488, 44489, 44490, 44491, 44492

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