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A PARTICULAR TEST ELEMENT OF A FREE SOLVABLE LIE ALGEBRA OF RANK TWO

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ABSTRACT. We prove that a free solvable Lie algebra of solvability class 3 generated by two elements has test rank 1 by giving a particular test element.

1. Introduction. Let F be an n-generator Lie algebra. A set of elements $g_1, g_2, \dots, g_r$, $r \leq n$, is a test set if for every endomorphism φ of F the conditions $\varphi(g_i) = g_i$ for $i = 1, 2, \dots, r$ imply that φ is an automorphism. The test rank of F is minimal cardinality of a test set. A test element is a test set consisting of one element. Well known examples of test elements for free groups were described by Nielsen [**8**] and Turner [**11**]. Mikhalev and Yu in [**6**] described an algorithm to determine test elements of free algebras of rank two.

Recently, Chirkov and Shevelin [**1**] and Esmerligil and Ekici [**4**] have independently shown that all nontrivial elements of a commutant of a free metabelian Lie algebra of rank 2 are test elements. They have further proved that the test rank of a free metabelian Lie algebra of rank n is equal to $n-1$. In the case of free solvable Lie algebras the situation is different from the metabelian case. Roman'kov [**9**] showed that a free solvable group of rank 2 and class 3 has test rank 1, and he constructed a test element for such groups.

The purpose of this article is to construct a test element for free solvable Lie algebras of rank 2 and solvability class 3.

2. Preliminaries and notations. Let F be a free Lie algebra over a field K with free generating set $\{x, y\}$. By $\delta^i F$ we denote the ith term of the derived series of F. We fix the notation $L = F \,/\, \delta^3 F$ for the free solvable Lie algebra generated by the set $\{\overline{x}, \overline{y}\}$ of solvability class 3, where $\overline{x} = x + \delta^3 F$, $\overline{y} = y + \delta^3 F$. Let $\widetilde{x}$, $\widetilde{y}$ denote the cosets $\widetilde{x} = x + \delta^2 F$ and $\widetilde{y} = y + \delta^2 F$. We know from [**2, 3**] that the universal

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