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POSITIVE SOLUTION OF MULTI-POINT BOUNDARY VALUE PROBLEM FOR THE ONE-DIMENSIONAL P-LAPLACIAN WITH SINGULARITIES

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ABSTRACT. In the paper, we get positive solutions of the following multi-point singular boundary value problem with p-Laplacian operator

$$\begin{cases} (\phi_p(u'))' + q(t)f(t,u,u') = 0 \quad 0 < t < 1, \\ u(0) = \sum_{i=1}^{n} \alpha_i u(\xi_i), \qquad\qquad u'(1) = \sum_{i=1}^{n} \beta_i u'(\xi_i), \end{cases}$$

where $\phi_p(s) = |s|^{p-2}s, p > 1$; $\xi_i \in (0,1)$, $i = 1,2,\dots,n$, $0 \le \alpha_i$, $\beta_i < 1$, $i = 1,2,\dots,n$, $0 \le \sum_{i=1}^{n} \alpha_i$, $\sum_{i=1}^{n} \beta_i < 1$ and $f(t,u,u')$ may be singular at $u = 0$, $u' = 0$.

1. Introduction. In this paper we study the singular boundary value problem (BVP for short)

$$\begin{cases} (\phi_p(u'))' + q(t)f(t,u,u') = 0 \quad 0 < t < 1, \\ u(0) = \sum_{i=1}^{n} \alpha_i u(\xi_i), \qquad\qquad u'(1) = \sum_{i=1}^{n} \beta_i u'(\xi_i), \end{cases} \tag{1.1}$$

where $\phi_p(s) = |s|^{p-2}s$, $p > 1$; $\xi_i \in (0,1)$, $i = 1,2,\dots,n$, $0 \le \alpha_i$, $\beta_i < 1$, $i = 1,2,\dots,n$, $0 \le \sum_{i=1}^{n} \alpha_i$, $\sum_{i=1}^{n} \beta_i < 1$ and $f(t,u,u')$ may be singular at $u = 0$, $u' = 0$, $q(t) \in C[0,1]$. The singular differential boundary value problem arises in many branches of both applied and basic mathematics and it has been extensively studied in the literature, for details, we refer the reader to [**2**].

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