

A SECTOR ISOPERIMETRIC PROBLEM

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ABSTRACT. We find the sector of largest area that can be formed by taking curves of fixed length that begin at some point on the positive x axis and connecting their endpoints to the origin with radial lines. We also discuss a more general question.

1. Statement of problem. The standard isoperimetric result in two dimensions is that of all possible closed curves of fixed length, the circle is the one that encloses the greatest area. See, for instance, the article [2] and its references. In this paper we are interested in looking not at a closed curve but at an open curve of fixed length s which we will assume starts at some point $(r, 0)$ on the x axis. We want to maximize the area of the sector created by connecting its endpoint to the origin. So the idea is to find which curve maximizes the area shown in Figure 1.

We will begin by assuming that the curve has length $s = 1$ and starts at the point $(1, 0)$ and show later the result for arbitrary s and r values which turns out to be no more difficult.

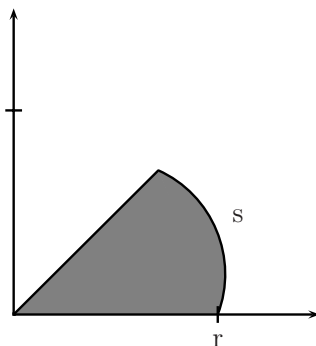


FIGURE 1. Maximize the shaded region.

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