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PERTURBATIONS IN THE SPEISER CLASS

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ABSTRACT. In this paper we study perturbations of maps from a family of expanding entire functions from the Speiser class. Maps in this family, which we denoted by $\mathcal{H}$, have the form $f_a(z) = \sum_{j=0}^{n} a_j e^{(j-k)z}$ where $0 < k < n$ and $a = (a_0, \dots, a_n) \in \mathbf{C}^{n+1}$ is a parameter. Using a known result of Eremenko and Lyubich about structural stability of such maps, perturbation theory (Kato-Rellich theorem) and research of Urbański and Zdunik on perturbations in the exponential family, we shall prove that the Hausdorff dimension of the set of points in the Julia set having nonescaping orbits depends analytically on the parameter $a \in \mathbf{C}^{n+1}$.

1. Introduction. The long-term study of dynamical systems directed many authors work toward the investigation of the dynamics of *families* of mappings. The most popular examples of families of transcendental entire functions of finite singular type include the one-parameter *exponential* family $\{ae^z\}$, the one-parameter *sine family* $\{a \sin z\}$, with $a \in \mathbf{C}$ or the generalized 2-parameter *cosine* family $\{ae^z + be^{-z}\}$ with $(a, b) \in \mathbf{C}^2$.

In this paper we continue our study of the dynamics of maps in the family $\mathcal{H}$ introduced in [**4**] and defined as follows. Let n and k be positive integers, let $a = (a_0, \dots, a_n) \in \mathbf{C}^{n+1}$ be a vector and let P_a, f_a be functions defined by the formulas

$$P_a(z) = a_n z^n + \cdots + a_1 z + a_0 \in \mathbf{C}[z],$$

$$f_a(z) = \frac{P_a(e^z)}{e^{kz}} = \sum_{j=0}^{n} a_j e^{(j-k)z}.$$

Then

$$\mathcal{H} = \Big\{ f_a : 0 < k < \deg P_a \text{ and } \delta_a > 0 \Big\}$$

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