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DERIVATIONS FROM SUBALGEBRAS OF OPERATOR ALGEBRAS: RESULTS AND PROBLEMS OLD AND NEW

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1. Introduction. Let $B \subseteq A$ be C^*-algebras. A linear map $\delta : B \to A$ is a derivation if $\delta(ab) = a\delta(b) + \delta(a)b$, for $a, b \in B$. If there is an element x of A for which $\delta(b) = (\mathrm{ad}\, x)(b) = xb - bx, b \in B$, we say that δ is *inner in* A, and *generated by* x. We will refer to any element x of A with this property as a *generator* of δ. If $B = A$, i.e., if δ is defined on all of A and maps A into itself, we will call δ a *derivation of* A. Because of its importance for what we will be discussing in the sequel, we recall that a *multiplier of* a C^*-algebra A is an element m of the enveloping von Neumann algebra A^{**} of A which multiplies A into itself ($mA \cup Am \subseteq A$). The set of all multipliers of A evidently forms a unital C^*-subalgebra of A^{**} which contains A as a closed, two-sided ideal and which is usually referred to as the *multiplier algebra of* A.

The purpose of this paper is to discuss some results and problems on derivations of operator algebras, with emphasis on those topics that have occupied the attention of the author for the past several years. Our dicussion centers around nine open problems that are posed at various places in the text. The main goal of the exposition is to discuss ideas and results that we hope motivate an interest in these problems, and which place them within the context of previous work on the subjects with which they deal. Because of this and the usual limitations of time and space, we have eschewed proofs, preferring instead to indicate precise references to places in the literature where proofs can be found by the interested reader.

2. Algebras with only inner derivations. The theory of derivations began to attract the attention of operator algebraists when I. Kaplansky proved in 1953 [**26**, Theorem 9] that all derivations of a type I von Neumann algebra are inner in the algebra. Thirteen years later, this was completed when S. Sakai [**43**], building on important preliminary work of R.V. Kadison [**22**], proved that a derivation of