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DUALITY TYPE RESULTS AND ERGODIC ACTIONS OF SIMPLE LIE GROUPS ON OPERATOR ALGEBRAS

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1. Introduction. Let G be a locally compact second countable group. We shall consider actions of G on a C^*-algebra A or on a von Neuman algebra M. By an action we shall mean a homomorphism $\alpha : G \to \text{Aut}(A)$ (respectively $\text{Aut}(M)$ of G into the group of all $*$-automorphisms of A (respectively M) such that the mapping $g \to \alpha_g(a)$ (respectively $g \to \alpha_g(m)$) is continuous for the topology of G and norm-topology of A (respectively ultraweak topology of M) for all $a \in A$ (respectively all $m \in M$).

In these cases we shall call the triple (A, G, α) (respectively (M, G, α) a C^*-dynamical system (respectively W^*-dynamical system). A W^*-dynamical system is called ergodic if the fixed point algebra $M^\alpha = \{m \in M | \alpha \in M | \alpha_g(m) = m, \text{ for all } g \in G\}$ is trivial, i.e., $M^\alpha = C \cdot 1$. A C^*-dynamical system (A, G, α) with the property that A^α is trivial will be called weakly ergodic. In the C^*-case, even in the classical, abelian case, further notions such as minimality and topological transitivity are to be considered. In order to give the noncommutative formulation of the above two notions, some notations are required.

A C^*-subalgebra $B \subset A$ is called a hereditary subalgebra of A if its positive part B_+ is a hereditary subcone of A_+ (i.e., for $x \in A_+, y \in B_+$ and $x \le y$ it follows that $x \in B_+$).

If (A, G, α) is a C^*-dynamical system, we denote by $\mathcal{H}^\alpha(A)$ the collection of all nonzero globally α-invariant hereditary subalgebras of A. Following [**6**, Definition 2.1] the action α is called topologically transitive if, for every B_1, $B_2 \in \mathcal{H}^\alpha(A)$ it follows that $B_1 \cdot B_2 \neq 0$.

With the generic name of ergodicity for all the notions formulated above, we shall investigate the following

PROBLEM. Let (A, G, α) be an ergodic C^*-dynamical system (respectively (M, G, α), an ergodic W^*-dynamical system). If $H \subset G$ is a closed subgroup, when is H ergodic on A (respectively M)?

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