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THE SUPPORTS OF MEASURES ASSOCIATED WITH ORTHOGONAL POLYNOMIALS AND THE SPECTRA OF THE RELATED SELF-ADJOINT OPERATORS

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Dedicated to Wolfgang J. Thron on the occasion of his 70th birthday

ABSTRACT. An elementary approach is given to prove Blumenthal's theorem describing the support of measures associated with orthogonal polynomials on the real line in case the recurrence coefficients associated with these polynomials tend to finite limits. Then the known approach using H. Weyl's theorem on compact perturbations of self-adjoint operators to Blumenthal's theorem is presented. Finally, using Weyl's theorem, Geronimus's result on the support is discussed when the recurrence coefficients with subscripts having the same residue $(\bmod k)$ have finite limits. Instead of the usual approach of using continued fractions, the Hardy class H^2 is used to determine the spectrum of the self-adjoint operator arising in the study of this support.

1. Introduction. In what follows, the word measure will always refer to a positive Borel measure on the real line such that its support is an infinite set and all its moments are finite. Here the support of a measure α is the smallest closed set whose complement has α-measure zero and is denoted by $\operatorname{supp}(\alpha)$, and the n-th moment of α is defined as

$$\int_{-\infty}^{\infty} x^n \, d\alpha(x), \qquad n \geq 0.$$

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