ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 21, Number 1, Winter 1991

A SEQUENCE OF BEST PARABOLA THEOREMS FOR CONTINUED FRACTIONS

LISA JACOBSEN AND DAVID R. MASSON

In honor of W.J. Thron on his 70th birthday

1. Introduction. The history of the parabola theorems dates back to 1940 when Scott and Wall published the first, simple version [**5**]. Our paper is based on the following beautiful generalization by Thron from 1958 [**6**]:

Theorem A. *Let* $-\pi/2 < \alpha < \pi/2$ *be a fixed number,* $\{g_n\}_{n=0}^{\infty}$ *be a fixed sequence with* $0 < g_0 \le 1$, $0 < g_n < 1$ *for* $n \ge 1$ *and*

$$\sum_{k=0}^{\infty} \prod_{n=1}^{k} \left(\frac{1}{g_n} - 1 \right) = \infty, \tag{1.1}$$

and let

$$P_{\alpha,n} = \{z \in \mathbf{C} : |z| - \operatorname{Re}(ze^{-i2\alpha}) \le 2g_{n-1}(1-g_n)\cos^2\alpha\} \tag{1.2}$$

for $n = 1, 2, 3, \ldots$ *. Finally, let* $K(a_n/1)$ *be a continued fraction with*

$$0 \ne a_n \in P_{\alpha,n} \text{ for all } n \in \mathbf{N}. \tag{1.3}$$

Then $K(a_n/1)$ *converges if and only if*

$$\sum_{n=1}^{\infty} \prod_{k=1}^{n} |a_k|^{(-1)^{n+k+1}} = \infty. \tag{1.4}$$

Comment 1. The conclusion in Theorem A is really just one of several proved in [**6**]. For instance, if (1.3) holds, then

Received by the editors on October 6, 1988, and in revised form on November 28, 1988.