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ON THE VORTEX SOLUTIONS OF SOME NONLINEAR SCALAR FIELD EQUATIONS*

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1. Introduction. Complex scalar nonlinear evolution equations have been used to model the dynamics of quantum structures. An example is the dynamics of quantum vortices in the theory of superfluids [**2, 1**]. In [**3**] a study is made of the effective dynamics of interacting vortices given nonlinear Schroedinger (NLS), Klein-Gordon (NLKG), and heat equations (NLH) as exact dynamics. The equations of evolution are

$$\begin{aligned} -i\psi_t &= \Delta\psi + (1-|\psi|^2)\psi \qquad &\text{(NLS)}\\ \psi_{tt} &= \Delta\psi + (1-|\psi|^2)\psi \qquad &\text{(NLKG)}\\ \psi_t &= \Delta\psi + (1-|\psi|^2)\psi \qquad &\text{(NLH)} \end{aligned}$$

where $\psi : \mathbf{R}^2 \times \mathbf{R}_+ \to \mathbf{C}$.

Vortex solutions of these equations are obtained in the form:

$$\psi_n(r,\theta) = U_n(r)e^{in\theta} \tag{1}$$

where (r,θ) denotes polar coordinates in $\mathbf{R}^2$, and $U_n(r)$ satisfies:

$$-\Delta_r U + n^2/r^2 U - (1-|U|^2)U = 0 \tag{2}$$

$$U(0) = 0 \quad \text{and} \quad U(r) \to 1 \quad \text{as } r \to \infty. \tag{3}$$

Here, $\Delta_r = \partial_{rr} + 1/r\partial_r$ is the radial Laplacian in $\mathbf{R}^2$. It can be shown [**3**] that a solution of (2)–(3) has the asymptotic behavior

$$U_n(r) \approx ar^n[1 - r^2/4(n+1)] \quad \text{as } r \to 0 \tag{4a}$$

$$1 - n^2/2r^2 \quad \text{as } r \to \infty. \tag{4b}$$

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