

## ALMOST PERIODICITY AND DEGENERATE PARABOLIC EQUATIONS

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**1. Introduction and results.** In the present paper we are interested in the almost periodic solutions to degenerate parabolic problems with special emphasis on the case of Stefan or porous media equations. We recall that the case of nondegenerate parabolic problems has been studied in the framework of nonlinear semigroups theory in [3] (for the case of linear or nonlinear variational inequalities see also [2]).

The central tool in this paper is Haraux's theorem on the existence of almost periodic solutions to abstract first order (in  $t$ ) equations with  $m$ -monotone operators, which are not uniformly monotone.

We give now the precise framework of this paper.

For the definition of (weakly) almost periodic functions in abstract spaces, we refer to [1, p. 1]. Let  $f(t)$  be a function in  $L^p_{\text{loc}}(R; X)$ ,  $p \geq 1$ , and consider the function

$$g(t) = \int_0^1 f(t+s) ds$$

from  $R$  to  $L^p(0,1; X)$ ; we say that  $f$  is  $S^p$ -(weakly) almost periodic (bounded) in  $X$  if  $g(t)$  is (weakly) almost periodic (bounded) in  $L^p(0,1; X)$ .

Let  $\Omega \subset R^N$  be a bounded open set with smooth boundary  $\Gamma$ ; consider the degenerate parabolic problem

$$(1.1) \quad D_t u - \Delta \beta(u) \ni f, \quad \beta(u)|_{\Gamma} = 0$$

where  $\beta$  is an increasing locally Lipschitz continuous function on  $R$  with  $\beta(0) = 0$ .

The Cauchy problem for (1.1) has been studied by H. Brézis, [4], as an application of general results on nonlinear semigroups, choosing as a Hilbert space the space  $H^{-1}(\Omega)$ .

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