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FOCAL SUBFUNCTIONS AND SECOND ORDER DIFFERENTIAL INEQUALITIES

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1. Introduction. In this paper we are interested in the differential equation

$$y'' = f(x, y, y'), \tag{1.1}$$

along with the "right focal" and "conjugate" boundary conditions (BC's), denoted respectively by

$$y(x_1) = y_1, \qquad y'(x_2) = y_2 \tag{1.2R}$$

and

$$y(x_1) = y_1, \qquad y(x_2) = y_2, \tag{1.2C}$$

where $x_1 < x_2, x_1, x_2 \in I$, an interval in $\mathbf{R}$, and $y_1, y_2 \in \mathbf{R}$ are arbitrary.

The BC's of the above type for equation (1.1) have been considered by several authors and for a variety of results concerning these problems, reference may be made to the papers [**1**–**10**] and to some of the other references contained therein.

For the sake of convenience we label the hypotheses that we use as follows:

A. f is continuous on $I \times \mathbf{R}^2$.

UC. Solutions of conjugate boundary value problems (BVP's) of (1.1), if they exist, are unique on I (that is, $y(x)$, $z(x)$ are solutions of the BVP (1.1), (1.2C) for arbitrary x_1, x_2 in I, $x_1 < x_2$ and $y_1, y_2 \in \mathbf{R}$ implies $y(x) \equiv z(x)$ on $[x_1, x_2]$).

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