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A SWITCH IN NODAL STRUCTURE IN COUPLED SYSTEMS OF NONLINEAR STURM-LIOUVILLE BOUNDARY VALUE PROBLEMS

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ABSTRACT. It is well known that the bifurcating nontrivial solutions to nonlinear Sturm-Liouville boundary value problems may be globally distinguished via the nodal structure of solutions. We demonstrate in this article that such is not necessarily the case for appropriate coupled multiparameter systems of such problems. Specifically, we give a calculable condition for the existence of a continuum of nontrivial solutions to such a system where the nodal structure of solution components is not preserved.

1. Introduction. Nonlinear Sturm-Liouville boundary value problems and associated systems arise frequently in mathematical analysis and applications. Consequently, there has been substantial interest in a detailed understanding of the solution sets to these problems, and a great deal of information has been obtained in the case of a single equation. For instance, consider the problem

$$-(p(t)x'(t))' + q(t)x(t) = \lambda(r(t)x(t) + f(t, x(t))) \tag{1.1}$$

$$\begin{aligned} \alpha_1 x(a) + \alpha_1' x'(a) &= 0 \\ \alpha_2 x(b) + \alpha_2' x'(b) &= 0, \end{aligned} \tag{1.2}$$

where $t \in [a, b]$ and $(|\alpha_1| + |\alpha_1'|)(|\alpha_2| + |\alpha_2'|) > 0$. In addition, we require that $p \in C^1[a, b]$ with $p(t) > 0$ on $[a, b]$, that $q, r \in C[a, b]$ with $r(t) > 0$ on $[a, b]$, and that $f : [a, b] \times \mathbf{R} \to \mathbf{R}$ is continuous with $\lim_{s \to 0} \frac{f(t,s)}{s} = 0$ uniformly for $t \in [a, b]$. In this situation, as is well known, there is a sequence

$$\lambda_1 < \lambda_2 < \cdots < \lambda_n \to +\infty$$

of simple eigenvalues for the problem

$$-(pw')' + qw = \lambda rw \tag{1.3}$$

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