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COMMUTATIVITY OF COONS AND TENSOR PRODUCT OPERATORS

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ABSTRACT. We show under what conditions Coons type surface approximation operators and tensor product approximation commute. An application is given for Bézier surfaces.

Definitions. We first define a *Coons patch*: Consider a surface patch $\mathbf{s}(u,v)$, which is a continuous map of the unit square into $\mathbf{R}^3$. We can define its (bilinearly blended) Coons approximation by

$$\begin{aligned} C\mathbf{s}(u,v) = {} & (1-u)\mathbf{s}(0,v) + u\mathbf{s}(1,v) \\ & + (1-v)\mathbf{s}(u,0) + v\mathbf{s}(u,1) \\ & - (1-u,u)\begin{pmatrix} \mathbf{s}(0,0), \mathbf{s}(0,1) \\ \mathbf{s}(1,0), \mathbf{s}(1,1) \end{pmatrix} \begin{pmatrix} 1-v \\ v \end{pmatrix}. \end{aligned} \tag{1}$$

This Coons patch interpolates to all four boundary curves of $\mathbf{s}$; in fact, it only depends on data from the boundary curves. For more details, see [**1** or **6**].

Let us next define a *tensor product surface*: Let $\mathbf{x}(t)$ be a curve, i.e., a continuous map of the unit interval into $\mathbf{R}^3$. We can define an approximation to it by

$$A\mathbf{x}(t) = \sum_{i=0}^{m} \mathbf{x}_i A_i(t), \tag{2}$$

where $\mathbf{x}_i = \mathbf{x}(t_i)$ for $0 = t_0 \le t_1, \dots, \le t_m = 1$. The $A_i(t)$ are univariate functions; they determine the nature of the approximation scheme. A second such scheme might be of the form

$$B\mathbf{x}(t) = \sum_{j=0}^{n} \mathbf{x}_j B_j(t).$$

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