ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 2, Spring 1992

UNCONDITIONALLY CONVERGING AND COMPACT OPERATORS ON c_0

ELIZABETH M. BATOR

ABSTRACT. It is shown that if Y is a Banach space, then c_0 imbeds in Y if and only if for every infinite dimensional Banach space X, there exists a noncompact operator $T : X \to Y$. In order to prove this, we first examine properties of operators on c_0, showing that if $T : c_0 \to X$ is a noncompact operator, then there exists a subspace Z of c_0 such that Z is isomorphic to c_0 and $T|_Z$ is an isomorphism.

The Josefson-Nissenzweig Theorem states that if X is an infinite dimensional Banach space, then there exists a weak*-null norm-1 sequence (x_n^*) in X^*. It is easy to see that, for such a sequence, $T : X \to c_0$, given by $T(x) = (x_n^*(x))$ is a noncompact operator. A natural problem then is to characterize the Banach spaces Y such that for every infinite dimensional Banach space X, there exists an operator $T : X \to Y$ such that T is noncompact. The goal of this paper is to show these Banach spaces are precisely those which contain isomorphic copies of c_0. Along the way we will examine properties of operators on c_0. Many properties of operators on c_0 can be determined by considering c_0 as a space of continuous functions on a locally compact Hausdorff space that vanish at infinity. For instance, one can modify the proof of Corollary IV.2.17 of [**3**] to get a corresponding result for c_0. The proof of this involves representing measures of such operators. Our investigation of these operators requires only a basic study of non relatively compact subsets of l^1.

All terms not defined in this paper can be found in [**2,3**]. If X is a Banach space, we denote the closed unit ball of X by B_X. Let X be a Banach space. Let (x_n^*) be a bounded sequence in X^* equivalent to

Received by the editors on February 1, 1989, and in revised form on July 12, 1989.
1980 AMS *Mathematics subject classification.* Primary 28B05, 46G10.
Keywords and phrases. Unconditionally converging operators, compact operators, Banach space.
Partially supported by a Faculty Research Grant from the University of North Texas.