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EXISTENCE THEORY FOR A STRONGLY DEGENERATE PARABOLIC SYSTEM

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ABSTRACT. An existence theory is established for the system $a\varphi_t = \operatorname{div}(\sigma(u)\nabla\varphi)$, $bu_t = \operatorname{div}(k(u)\nabla u) + \sigma(u)|\nabla\varphi|^2$ in a bounded domain of $\mathbf{R}^N$ coupled with initial-boundary conditions. We only assume that σ, k are positive, and thus the system may become degenerate as u goes to infinity. As a result, solutions of the problem display new phenomena that cannot be incorporated into the classical weak formulation. A generalized notion of a solution developed in [**9, 10**] is employed to handle the problem.

1. Introduction. Let Ω be a bounded domain in $\mathbf{R}^N$ with smooth boundary $\partial\Omega$ and T a positive number. Set $Q_T = \Omega \times (0,T)$, $S_T = \partial\Omega \times (0,T)$. Consider the following initial-boundary-value problem:

$$a\varphi_t = \operatorname{div}(\sigma(u)\nabla\varphi) \qquad \text{in } Q_T \tag{1.1a}$$

$$bu_t = \operatorname{div}(k(u)\nabla u) + \sigma(u)|\nabla\varphi|^2 \qquad \text{in } Q_T \tag{1.1b}$$

$$\varphi = \bar{\varphi} \qquad \text{on } S_T \tag{1.1c}$$

$$u = 0 \qquad \text{on } S_T \tag{1.1d}$$

$$u = u_0, \qquad \varphi = \varphi_0 \qquad \text{on } \Omega \times \{0\}. \tag{1.1e}$$

Here a and b are given positive constants and $\sigma(u), k(u)$ are known functions of u.

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