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PRODUCTS OF SYMPLECTIC GROUPS ACTING ON ISOTROPIC SUBSPACES

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ABSTRACT. Let A be a finite dimensional commutative semisimple algebra over a field k, and let (V, B) be a finitely generated symplective space over A. We examine the action of the symplectic group $\mathrm{Sp}_A(V, B)$ on the set of B'-isotropic k-subspaces of V, where $B' = \phi \circ B$ is the k-symplectic form induced by a 'trace' map $\phi : A \to k$. The case of A being a field was studied earlier and here we consider the case where A has several simple components. The orbits are completely classified when $A = k \times k$ and for maximal B'-isotropic subspaces when $\dim_k A = 3$; the number of orbits of maximal B'-isotropic subspaces is infinite if $\dim_k A \geq 4$ and k is infinite.

1. Introduction. In [**6**] and [**3**] we studied the action on Grassmannians of products of general linear groups defined over extension fields of the base field. In [**4**] this work was extended to the case of a symplectic group defined over an extension field of the base field acting on isotropic subspaces of a symplectic space. The present paper examines the case of a product of symplectic groups acting on isotropic subspaces, i.e., we replace the extension field by a finite dimensional commutative semisimple algebra.

In more detail, the problem is the following. We have a finite dimensional commutative semisimple algebra A over a field k, a finitely generated faithful A-module V and an A-valued regular symplectic form B on V. By suitably choosing a k-linear functional $\phi : A \to k$ (see Section 2), we form the k-valued symplectic form $B' = \phi \circ B$ and look at the action of the symplectic group $\mathrm{Sp}_A(V, B)$ on B'-isotropic k-subspaces of V. The problem is to determine when the number of orbits is finite and to classify them. In group theoretic terms, if $A = k_1 \times \cdots \times k_p$ where each k_i/k is a finite extension of fields, this

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