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WELL-POSED OPTIMIZATION PROBLEMS AND A NEW TOPOLOGY FOR THE CLOSED SUBSETS OF A METRIC SPACE

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ABSTRACT. We provide a further analysis of the bounded proximal topology, recently defined in the setting of minimization problems and then studied by the present authors in a unifying article on hyperspace topologies. We exhibit its main topological properties, and we compare it with other hyperspace topologies. We further consider this topology in the context of minimization problems, specifically with respect to problems that are well-posed in the generalized sense (g.w.p.). It is shown that the solution set of such a minimum problem can be recovered from a sequence of level sets of approximating functions and that nearby problems to a given g.w.p. convex function will necessarily have a solution if and only if the underlying space is reflexive. On the other hand, nearby problems need not be g.w.p., even if they have unique minimizers.

1. Introduction. When dealing with minimization problems we have to consider sets that represent not only constraint sets but also functions, as identified with their epigraphs. Thus, topologies on the closed sets of a metric space (called *hyperspace topologies* [**30**]), are a fundamental tool in some aspects of optimization, as for instance in stability analysis. But the best known one—the Hausdorff metric topology [**18, 26**]—is not usually well-suited for this analysis because it fails to work well when sets under analysis are unbounded. The first attempt at overcoming this difficulty was made by using the notions of topological Lim sup and Lim inf of a sequence (or net) of sets [**27**, Section 29]. A sequence $\langle A_n \rangle$ of closed sets is declared *Painlevé-Kuratowski convergent* to the set A if, at the same time, $A = \operatorname{Lim\,sup} A_n$ and $A = \operatorname{Lim\,inf} A_n$. When the metric space X

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