ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 1, Winter 1994

ASYMPTOTICS AND OSCILLATION FOR FIRST ORDER NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS

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Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. The asymptotic behavior of all nonoscillatory solutions of some linear first order neutral delay differential equations is studied. A solution to one conjecture by Grammatikopoulos, Grove and Ladas is given and part of a second conjecture is proved. A test for oscillation in terms of the coefficients, the delay and the advance is given for neutral equations of mixed type.

1. Introduction. In this paper we deal with the first order linear neutral functional differential equation

$$\frac{d}{dt}[y(t) + py(t-\tau)] + qy(t-\sigma) = 0 \tag{1}$$

for $t \geq t_0$ where $q \neq 0$, p, τ and σ are real numbers.

From the point of view of applications, NFDEs appear as models of electrical networks which contain lossless transmission lines. Such networks arise, for example, in high speed computers where lossless transmission lines are used to interconnect switching circuits (see [**2, 11**]). Concerning existence, uniqueness and continuous dependence of solutions, see Hale [**7**] and Driver [**3, 4**].

As usual, a solution of equation (1) is called oscillatory if it has arbitrarily large zeros and nonoscillatory if it is eventually positive or negative.

Recently, Grammatikopoulos, Grove and Ladas in [**5**] studied asymptotic behavior of nonoscillatory solutions of NFDEs (1) under various

Received by the editors on September 2, 1993
AMS 1991 *Mathematics Subject Classification.* 34K25, 34K40.
Key Words. Neutral equation, asymptotic behavior, oscillation.
Part of this paper was written while the first and second authors were visiting the Applied Mathematics Institute at the University of Alberta.
Research of the second author supported in part by the Canada-China Scholarly Exchange Program.