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SURJECTIVITY RESULTS FOR COMPACT PERTURBATIONS OF STRONGLY MONOTONE OPERATORS IN BANACH SPACES

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ABSTRACT. The operator equation $Au + Lu + cFu = h$ is studied in a Banach space X and its dual space. The operators L, F are compact and A is strongly monotone. Degree arguments are used to show the existence of solutions of the equation and extension of the results in [**2, 3**] are established.

1. Introduction. In the following, the symbols R and R_+ denote the sets $(-\infty,\infty)$ and $[0,\infty)$, respectively. X stands for a real Banach space having a Schauder basis $\{x_i\}$. Without loss of generality, we will assume that X is normed so that both X and X^* are locally uniformly convex and $||x_i|| = 1$, $i = 1, 2, 3, \dots$. Referring to the book ([**4**, pages 25, 272]) there exists a constant $M \geq 1$, independent of n, such that

$$\left\|\sum_{i=1}^{n} a_i x_i\right\| \leq M \left\|\sum_{i=1}^{\infty} a_i x_i\right\|, \qquad n = 1, 2, 3, \dots \tag{1}$$

and

$$\sup\{|\langle \Phi, x\rangle| : ||x|| \leq 1, x \in E_k^{\perp}\} \to 0, \qquad k \to \infty \tag{2}$$

for each $\Phi \in X^*$. Here

$$E_k = \operatorname{span}\{x_1, x_2, \dots, x_k\}$$

and

$$E_k^{\perp} = \operatorname{span}\{x_{k+1}, x_{k+2}, \dots\}.$$

Lemma 1. *Let $L : X \to X^*$ be a compact mapping. Then*

$$\lim_{k\to\infty} \sup\{|\langle Lf, f\rangle| : f \in E_k^{\perp}, ||f|| \leq 1\} = 0.$$

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