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A RESTRICTION ON THE FIRST SUCCESSOR CARDINAL WHICH IS JONSSON

ROBERT MIGNONE

ABSTRACT. This paper investigates Jonsson cardinals as successors of singular cardinals. In particular, Shelah's theory of possible cofinalities is used to show that the first successor cardinal which is Jonsson cannot be the successor of a singular cardinal whose cofinality is measurable.

Viewing an algebra as a nonempty set together with countably many finite argument functions on the set, an algebra is said to be Jonsson if any proper subalgebra must have smaller cardinality.

When the cardinal κ is taken as the nonempty set, then κ is said to bear a Jonsson algebra if countably many finite argument functions exist on κ, which cannot have their respective domains and ranges restricted to a proper subset of κ whose cardinality is κ. Otherwise, κ is said to be a Jonsson cardinal.

Jonsson's problem concerns which cardinals are Jonsson cardinals. For more background see [**2**].

Charting some of the classical and more recent results relating to Jonsson's problem in ZFC yields the following:

$\aleph_0, \aleph_1, ..., \aleph_n, ...$ are not Jonsson cardinals, see [**2**].

$\aleph_\omega$ open.

$\aleph_{\omega+1}$ is not a Jonsson cardinal (Shelah), see [**4**] or [**6**].

$\vdots$

$\aleph_{\alpha+1}$ is not a Jonsson cardinal if $\aleph_\alpha$ is regular,

(Tryba, Woodin) see [**1**] or [**7**].

$\vdots$

$\aleph_\beta$ is the first Jonsson cardinal implies either $cf(\aleph_\beta) = \omega$

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