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SELECTIONS OF MAPS WITH NONCLOSED VALUES AND TOPOLOGICALLY REGULAR MAPS

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ABSTRACT. We introduce the concept of a topologically regular map as a map with homeomorphic fibers, whose multivalued inverse map is continuous with respect to the Fréchet metric. Using E. Michael's selection theorem, we prove that every topologically regular map with the fibers homeomorphic to $[0,1]$, of a locally σ-compact metric space onto a perfectly normal space, is a locally trivial bundle.

Let $p : E \to B$ be a continuous map. We are interested in finding conditions which guarantee that p is a locally trivial fibration. Clearly, a necessary condition is that the fibers $p^{-1}(b)$, $b \in B$, are homeomorphic.

Denote by $\exp^M(E)$ the class of all closed subsets of the topological space E which are homeomorphic to a fixed topological space M. Usually the set $\exp^M(E)$ is equipped with the Vietoris topology, which in the case when E is metrizable and M is compact coincides with the topology, induced by the Hausdorff metric. However, such a topology doesn't take into account the uniqueness (up to homeomorphism) of the elements of $\exp^M(E)$. We shall introduce a metric in $\exp^M(E)$ which will not have this defect:

Definition 1. The *Fréchet distance* between two homeomorphic closed subsets A and B of the metric space E is the infimum of all $\varepsilon > 0$ for which there exists a homeomorphism $h : A \to B$ (or $h : B \to A$) which doesn't move points for more than ε, i.e., h is an ε-move.

In [**3**] this distance was called the *homeomorphic distance*. In the present paper we chose the new term having in mind the analogy

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