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STABILITY PROPERTY AND PHASE SPACE

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1. Introduction. Recently Murakami and Yoshizawa [**16**] have discussed the relationship between the BC-stability and the ρ-stability in a class of functions bounded by a priori bound for a functional differential equation defined on a phase space X with a seminorm $||\cdot||_X$. The BC-stability means that the solution remains small if the initial function is small with respect to the BC-norm, $|\cdot|_{(-\infty,0]}$, while the ρ-stability corresponds to the ρ-metric:

$$\rho(\varphi) := \sum_{k=1}^{\infty} 2^{-k} \frac{|\varphi|_{[-k,0]}}{1+|\varphi|_{[-k,0]}},$$

where $|\varphi|_I := \sup_{s\in I} |\varphi(s)|$ for an interval I.

The situation above is rather complex; there appear three metrics, and the restriction to the class of functions bounded by a bound will be observed to effect on these metrics.

The purpose of this paper is to clarify the relationships between these metrics and to give a unified aspect on the concepts of the stability by allowing more flexibility in the choice of the phase space. Haddock and Hornor [**7**] have introduced the concept of the H-stability related with a fading memory subspace H of X, see the latter, Example 3, for the definition. Our idea will show that this turns out to be a problem of the choice of the suitable phase space.

Consider the equation

$$\text{(E)} \qquad \dot{x}(t) = f(t, x_t),$$

where $f(t,\varphi)$ is defined and continuous on $[0,\infty)\times X$ for a phase space X. Then it will be easier to see the existence of a solution in a space with a weaker topology if $f(t,\varphi)$ endows an adequate regularity there. However, the weaker the topology of the space is, the more meager

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