ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 1, Winter 1995

A NEW ANGLE ON STURM-LIOUVILLE PROBLEMS

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Introduction. The problem under study takes the form

$$-(py')' + qy = \lambda ry, \tag{1}$$

where

$$p > 0, \quad r > 0, \quad 1/p, q, r \in L_1([0,1]), \mathbf{R}),$$

subject to boundary-conditions of the form

$$y(0)\cos\beta_0 = (py')(0)\sin\beta_0, \qquad 0 \le \beta_0 < \pi \tag{2}$$

and

$$(a\lambda + b)y(0) = (c\lambda + d)(py')(0), \tag{3}$$

where

$$0 \ne (a,b,c,d) \in \mathbf{R}^4 \quad \text{and} \quad e = ad - bc. \tag{4}$$

Extensive bibliographies for this problem can be found in Walter [**8**] and Fulton [**5**]. Most of the cited work deals with completeness and expansion theory in $L_2[0,1] \oplus \mathbf{C}$. Here we consider Sturm (oscillation, comparison, etc.) theory for three cases:

(I) $c = 0 \ne d$, $e \ge 0$ (also discussed by Reid [**7**] via different methods);

(II) $c \ne 0$, $e > 0$ (joint work with P.J. Browne and K. Seddighi [**4**]);

(III) $c \ne 0$, $e < 0$ (joint work with P.J. Browne).

Further details for these and other cases (e.g., with both end conditions λ dependent, indefinite r, etc.) will appear elsewhere.

(I) *The simplest case.* We remark that this case includes the Sturm-Liouville (λ-independent end condition) one where $a = 0$. We define θ by means of the differential equation

$$\theta' = (1/p)\cos^2\theta + (\lambda r - q)\sin^2\theta$$

Received by the editors on July 28, 1992.