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THE FLAT DIMENSION OF MIXED ABELIAN GROUPS AS E-MODULES

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1. Introduction. In 1989, Faticoni and Goeters [**6**] constructed a large variety of torsion-free abelian groups of finite rank which were flat as modules over their endomorphism rings, while Albrecht [**2**] gave examples of infinite rank groups with the same property in 1990. In his survey talk presented at the 1989 University of Connecticut abelian groups conference, R.S. Pierce proposed the general problem of E-flatness as a worthy study—particularly that of computing the flat dimension $(fd_{E(G)}(G))$ of an abelian group as a left module over $E = E(G)$. The only known major results along these lines were the theorem of Richman and Walker [**12**] proving that every reduced torsion group is E-flat and the theorem of Arnold [**3**] giving a criterion for a completely decomposable group to be E-flat.

In 1991, Vinsonhaler and Wickless [**15**] constructed finite rank, completely decomposable groups $\{G_n \mid 0 \leq n \leq \infty\}$ with $fd_E(G_n) = n$. Recently, Dugas and Faticoni [**7**] have announced a similar result employing a different method and constructing different kinds of torsion-free examples.

In this paper, we primarily study a class $\mathcal{G}$ of mixed abelian groups. The elements $G \in \mathcal{G}$ will be of finite torsion-free rank and embedded as a pure subgroup into ΠG_p, where G_p denotes the p-torsion subgroup of G. Furthermore, each $G \in \mathcal{G}$ will satisfy an additional requirement connected with the self-small property which was introduced in [**4**] by Arnold and Murley.

Our main result is that, for each $0 \leq n \leq \infty$, there exists $G \in \mathcal{G}$ with $fd_E(G) = n$. In the course of proving this, we show that, if $G \in \mathcal{G}$ has torsion-free rank n, then $fd_E(G) = fd_A(M)$. Here M is the algebra of $n \times n$ rational matrices and $A \subseteq M$ is a rational subalgebra associated with the group G. This latter result leads to a realization problem: Which subalgebras $A \subseteq M$ are associated with some $G \in \mathcal{G}$? We show

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