ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 3, Summer 1995

SMOOTH PARTITIONS OF UNITY AND APPROXIMATING NORMS IN BANACH SPACES

DAVID P. MCLAUGHLIN

1. Introduction. In the seminal paper [**1**] it is shown that C^k-smooth separable Banach spaces admit C^k-smooth partitions of unity (definitions to follow), but it is still an open question whether this result extends to nonseparable spaces; contributions to this question appear in [**15, 10, 3, 16** and **13**]. A survey of these and related results can be found in [**4**, Section 8.3] and we refer the reader to the notes and references therein.

Theorem 4 in [**15**] states that a reflexive Banach space X admits C^k-smooth partitions of unity whenever X admits an LUR norm which is C^k-smooth. Two observations motivate the main result of this paper: first, a reflexive C^k-smooth Banach space admits C^k-smooth partitions of unity; second, in general, a space with a C^k-smooth norm and an LUR norm will not have a norm which is both C^k-smooth and LUR. In fact, let us note that Asplund's averaging technique (cf. [**4**, Section 2.2.4]) for higher order smooth norms is in general not available, e.g., $c_0(\mathbf{N})$ has a C^∞-smooth norm and its dual has an LUR norm, but $c_0(\mathbf{N})$ has no LUR C^2-smooth norm [**8**]. (Note: the corresponding averaging result for WLUR is open; because the set of C^2-smooth norms is the first category, a Baire category arguments sheds no light on this question.)

We now present the main result of this paper.

Theorem 1. *Suppose a* WLUR *norm on a Banach space X can be uniformly approximated on bounded sets by equivalent C^{k+1}-smooth norms, where $k \in \mathbf{N} \cup \{\infty\}$. Then X admits C^k-smooth partitions of unity.*

Remarks a. Note that in this theorem we do not assume *a priori* that the space in question admits a mapping into some $c_0(\Gamma)$.

Received by the editors on October 1, 1993.