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# THE DIFFERENTIAL INVARIANTS OF PARTICLE LAGRANGIANS UNDER EQUIVALENCE BY CONTACT TRANSFORMATIONS

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ABSTRACT. We use Élie Cartan's method of equivalence to derive the structure equations of the integral $\int L(x, y^1, \ldots, y^m, dy^1/dx, \ldots, dy^m/dx)dx$ under the group of contact transformations for the case $m > 1$. These equations define a complete set of local differential invariants of the integral under contact transformations. We obtain a differential quadratic form and an associated system of frames which are intrinsic to $\int L\,dx$ and interpret our results from the standpoint of Finsler spaces. In the last section we explore some of the consequences of the structure equations.

**1. Introduction.** This paper extends results obtained by Robert Gardner and Robert Bryant in an unpublished work [**3**] on the problem of finding local differential invariants of the integral

$$\int L(x, y^1, \ldots, y^m, dy^1/dx, \ldots, dy^m/dx)\, dx$$

under the group of contact transformations for the case $m > 1$ with the assumption that $L$ is a regular Lagrangian. This problem was solved by Élie Cartan for the case $m = 1$ [**4**]; S.S. Chern [**6**] found the differential invariants of the integral

$$\int L(y^1, \ldots, y^m, dy^1/dx, \ldots, dy^m/dx)\, dx$$

where $L$ is positively homogeneous of degree one in the variables $p^1, \ldots, p^m$.

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