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CONDITIONAL FUNCTION SPACE INTEGRALS WITH APPLICATIONS

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ABSTRACT. In the theory of the conditional Wiener integral, the integrand is a functional of the standard Wiener process. In this paper we first consider a conditional function space integral for functionals of more general stochastic process and obtain an evaluation formula of the conditional function space integral. We then use this formula to derive the generalized Kac-Feynman integral equation and also to obtain a Cameron-Martin type translation theorem for our conditional function space integrals. These results subsume similar known results obtained by Chung and Kang, Park and Skoug, and Yeh for the standard Wiener process.

1. Introduction. Let $(C_0[0,T], \mathcal{B}(C_0[0,T]), m_w)$ denote Wiener space where $C_0[0,T]$ is the space of all continuous functions x on $[0,T]$ with $x(0)=0$. Many physical problems can be formulated in terms of the conditional Wiener integral $E[F|X]$ of the functionals defined on $C_0[0,T]$ of the form

$$F(x) = \exp\bigg\{ -\int_0^t \theta(s, x(s))\, ds\bigg\} \tag{1.1}$$

where $X(x) = x(t)$ and $\theta(\cdot,\cdot)$ is a sufficiently smooth function on $[0,T]\times \mathbf{R}$. It is indeed known from a theorem of Kac [**8**] that the function $U(\cdot,\cdot)$ defined on $[0,T]\times\mathbf{R}$ by

$$U(t,\xi) = \frac{1}{\sqrt{2\pi t}} \exp\bigg\{ -\frac{(\xi-\xi_0)^2}{2t}\bigg\} E[F(x(t)+\xi_0)|x(t) = \xi-\xi_0] \tag{1.2}$$

is the solution of the partial differential equation

$$\frac{\partial U}{\partial t} = \frac{1}{2}\frac{\partial^2 U}{\partial \xi^2} - \theta U \tag{1.3}$$

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