ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 27, Number 1, Winter 1997

ON HOMOGENEOUS COVERING CONGRUENCES

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Dedicated to Professor E. Hlawka on the occasion of his eightieth birthday

ABSTRACT. Two problems on covering congruences proposed by T. Cochrane and G. Myerson are either solved or reduced to a classical problem of Erdős.

T. Cochrane and G. Myerson [**2**] called a cover for the additive group $\mathbf{Z}^n$ any system of congruences

$$(1) \qquad \sum_{j=1}^{n} a_{ij}x_j \equiv a_{i0} \pmod{m_i}, \qquad 1 \le i \le r,$$

where

$$(2) \qquad (a_{i0}, a_{i1}, \dots, a_{in}, m_i) = 1, \qquad 1 \le i \le r,$$

such that every vector $\langle x_1, \dots, x_n \rangle \in \mathbf{Z}^n$ satisfies at least one of them and

$$(3) \qquad 1 < m_1 < \dots < m_r.$$

If $a_{i0} = 0$, $1 \le i \le r$, (1) is called a homogeneous cover.

We prefer to use the terms cover and homogeneous cover in a wider sense, without the condition (3), and we shall formulate the relevant results and problems in that way. The best known problem about covering congruences, due to Erdős [**3**], concerns the truth of the following statement.

E. *For every $c \in \mathbf{N}$ there exists a cover for $\mathbf{Z}$ with distinct moduli greater than c.*

Cochrane and Myerson proved the following lemma. Let $x \equiv a_i \pmod{m_i}$, $1 \le i \le r$, be a cover for $\mathbf{Z}$ in which moduli are distinct

Received by the editors on February 21, 1996.
1991 *Mathematics Subject Classification.* 11A07, 11B27.
Key words and phrases. Covering congruences.