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ON THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF A CLASS OF SELFADJOINT SECOND ORDER LINEAR SYSTEMS

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ABSTRACT. Suppose $P(x)$ is an $N \times N$ positive definite real matrix-valued C^1-function on $0 \leq x < \infty$ such that $P'(x)$ is also positive definite for x sufficiently large. We prove that if Trace$[P(x)] \to \infty$ as $x \to \infty$, then the second order linear system $y''(x) + P(x)y(x) = 0$ has a nontrivial solution which tends to zero in norm at infinity. Do all nontrivial solutions of the system tend to zero in norm at infinity? For this question we find a criterion. And applying this criterion we prove that if $P(x) = Q^2(x)$, where $Q(x)$ is a real symmetric matrix polynomials of degree ≥ 1, and with positive definite leading coefficient, then the question has an affirmative answer.

1. Introduction. In this paper we study the asymptotic behavior of the solutions of the following self-adjoint second order linear system

$$y''(x) + P(x)y(x) = 0, \tag{1.1}$$

where $P(x)$ is an $N \times N$ positive definite matrix-valued function on $[0, \infty)$, $y(x)$ is an $\mathbf{R}^N$-valued function, 0 is the zero vector in $\mathbf{R}^N$. We are interested in the questions of finding sufficient conditions which guarantee the existence of a nontrivial solution $y_0(x)$ of (1.1) such that $\lim_{x\to\infty} \|y_0(x)\| = 0$, where $\|y_0(x)\|$ is the norm of $y_0(x)$, and of finding sufficient conditions which guarantee that the norm of any nontrivial solution of (1.1) tends to zero in norm as x approaches infinity. We notice that, for the case $N = 1$, i.e., $P(x)$ in (1.1) is a scalar function, these questions had been studied by many mathematicians, notably Milloux, Hartman, Lazer, Meir, Willett and Wong (see [**1, 3, 5, 6**] and the references in these papers and book). In [**3**], for the case $N = 1$, Hartman used the Liouville transformation to transform (1.1) to a first order differential system, then he observed the related first order system and proved the Milloux theorem which says that if the

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