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QUASI-MEASURES ON COMPLETELY REGULAR SPACES

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ABSTRACT. Let X be a completely regular space. We give definitions of a Baire quasi-measure on X and a quasi-state on $C_b(X)$, the space of bounded, real-valued continuous functions on X. A representation theorem is developed for quasi-states on $C_b(X)$ in terms of Baire quasi-measures on X. We also define various notions of smoothness for quasi-measures and quasi-states, and then we furnish examples which demonstrate the different types of smoothness. Finally, by considering the space X to be embedded in its Stone-Čech compactification βX, the smoothness of a Baire quasi-measure μ on X is characterized by the behavior of $\bar{\mu}$, the corresponding Baire quasi-measure on βX.

1. Introduction. The theory of quasi-measures evolved from the study of certain nonlinear functionals (quasi-states) on commutative C^*-algebras. The goal here is to extend to completely regular spaces the theory of quasi-measures on compact Hausdorff spaces initiated by J. Aarnes [**1**]. For definitions and results regarding quasi-measures on compact Hausdorff spaces the reader is encouraged to consult [**1, 2**]. Our standard guide to work in topological measure theory is the survey paper by Wheeler [**11**].

One of the main goals of this paper is to develop a representation for quasi-states on $C_b(X)$ in terms of Baire quasi-measures on X. This is accomplished by generalizing results of Aarnes in [**1**] to completely regular spaces. We then state definitions of various types of smoothness for quasi-measures and quasi-states. Then, following work done by Varadarajan [**10**] for ordinary Baire measures, we demonstrate the connection between the smoothness of a Baire quasi-measure and the smoothness of its corresponding quasi-state.

Finally, using the techniques of Knowles [**6**], we characterize the smoothness of a Baire quasi-measure μ on X by the behavior of $\bar{\mu}$, the unique Baire quasi-measure on βX corresponding to μ. In Section 2

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