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BILINEAR INTEGRATION IN TENSOR PRODUCTS

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ABSTRACT. The integration of vector valued functions with respect to vector valued measures is studied in this paper. The resulting integral take its values in a tensor product space satisfying a mild separation condition. The relationship with bilinear integrals of R. Bartle and I. Dobrakov is examined. The present integral has the advantage of allowing a treatment of integration with respect to certain measures derived from spectral measures, that is not otherwise available.

0. Introduction. The notion of integrating vector valued functions with respect to vector valued measures has been treated by a number of authors [**1, 4**–**7, 2**]. If the indefinite integral takes its values in a tensor product space, then the special nature of topological tensor products can be exploited to yield an integration theory suitable for bilinear integration with respect to spectral measures. Integrals of this nature arise in the study of random evolutions with respect to operator valued measures [**10**].

As a guide to the sort of properties we are looking for, let $1 \le p < \infty$ and $X = Y = L^p([0,1])$, and consider $X \otimes Y$ as a dense subspace of $L^p([0,1]^2)$. Let $\{y_j\}_{j=1}^{\infty}$ be an unconditionally summable sequence in Y, and set $m(A) = \sum_{j \in A} y_j$ for each subset A of $\mathbf{N}$. Then m is a Y-valued measure. A *scalar valued* function $f : \mathbf{N} \to \mathbf{C}$ is m-integrable if and only if $\{f(j)y_j\}_{j=1}^{\infty}$ is unconditionally summable in Y. It is reasonable, therefore, that an X-valued function $f : \mathbf{N} \to X$ should be m-integrable in $L^p([0,1]^2)$ whenever $\{f(j) \otimes y_j\}_{j=1}^{\infty}$ is unconditionally summable in $L^p([0,1]^2)$.

Although this looks like a natural starting point for bilinear integration, it gives rise to some unusual features. For example, suppose that $1 \le p < 2$. Then there exists an unconditionally summable sequence $\{y_j\}_{j=1}^{\infty}$ in $L^p([0,1])$ and a bounded function $f : \mathbf{N} \to L^p([0,1])$

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